Sections 8.2, 9.3, and 10.4 Population Proportion

Sections 8.2 – Distribution of the Sample Proportion

We might be interested in

- the proportion of US males who have health insurance,
- the proportion of imported cars in the US,
- the proportion of Americans who have Type II diabetes,

P: Population Proportion.

$\widehat{P} = \frac{x}{n}$: Sample Proportion. $\widehat{P}$ is a point estimate of P, the population proportion.

Example 1: A sample of 200 married couples selected from throughout the United States showed that for 84 of the couples, both the husband and wife held full-time jobs. Use the sample results to estimate the proportion of all married couples in the United States for which both the husband and wife hold full-time jobs.

Solution: $\widehat{P} = \frac{x}{n} = \frac{84}{200} = 0.42$

The Sampling Distribution of the proportion-CLT

The $\mu_{\widehat{P}} = E\left(\widehat{P}\right) = P$ and the $\sigma_{\widehat{P}} = \sqrt{\frac{p(1-p)}{n}}$. For a simple random sample of size n from a population with a proportion, P, the sampling distribution of the sample proportion, $\widehat{P}$, is approximately normally distributed with mean, P, and standard deviation of $\sigma_{\widehat{P}} = \sqrt{\frac{p(1-p)}{n}}$, if **np(1 – p) ≥ 10**, i.e $\widehat{P} \sim N\left(P\,,\ \sqrt{\frac{P(1-P)}{n}}\right)$.

Example 2: A particular county in West Virginia has a 9% unemployment rate. To monitor the unemployment rate in that county, a monthly survey of 800 individuals is conducted by a state agency.

(a) What is the sampling distribution of $\widehat{P}$ when a sample size of 800 is used?

(b) In the sample of 800, what is the probability at least 64 people will be will be unemployed? **Solution (a) $\widehat{P}$ ~ N(0.09, 0.0101) (b) 0.8389**

Homework-Section 8.2: 15, 16 page 399

Sections 9.3 Confidence interval for a population proportion

Confidence interval for a proportion: $\widehat{P} \pm Z_{\frac{\alpha}{2}}\sqrt{\dfrac{\widehat{P}\left(1-\widehat{P}\right)}{n}}$

Example 1. In a Roper Organizational poll of 2,000 adults, 1,280 have money in regular saving accounts. Find a 95% confidence interval for the true proportion of adults who have money in regular saving accounts.

Solution. $\widehat{P} = \dfrac{x}{n} = \dfrac{1280}{2000} = 0.64$, (0.619, 0.661) **From TI-83/84**

We are 95% confident that the percentage of adults having money in the saving account is **61.9% to 66.1%.**

Determination of the Sample Size: Let E = $Z_{\frac{\alpha}{2}}\sqrt{\dfrac{p(1-p)}{n}}$

Solving for n gives us, n = $\left(\dfrac{Z_{\frac{\alpha}{2}}}{E}\right)^2 \widehat{P}$ (1 - $\widehat{P}$); If a preliminary estimate on $\widehat{P}$ is given use it, if not use $\widehat{P}$ = 0.5 .

Example 2. We want to estimate with a maximum error of 3%, the true proportion of VSU students who would like Friday classes during the summer semester and we want 96% confidence in our results. How many VSU students must we survey?

Solution. n = $\left(\dfrac{Z_{\frac{\alpha}{2}}}{E}\right)^2 \widehat{P}$ (1 - $\widehat{P}$)= $\left(\dfrac{2.0537}{0.03}\right)^2$(0.5)(0.5) = 1171.57 . So, we need to survey 1172 students.

Example 3. The current percentage of voters favoring the Republican nominee is 61%. A pollster is hired to determine the percentage of voters favoring the Republican nominee. If we require 99% confidence that the estimated value is within 2% of the true value, how large should the sample be?

Solution. n = $\left(\dfrac{Z_{\frac{\alpha}{2}}}{E}\right)^2 \widehat{P}$ (1 - $\widehat{P}$)= $\left(\dfrac{2.5758}{0.02}\right)^2$(0.61)(0.39)= 3946.01 . So, we need to survey 3,947 voters.

Homework-Section 9.3 20, 21, 26, 27, 29, and 31 pages 442-443

Section 10.4 Hypothesis Testing for a Population Proportion

Example 6. Mr. Dixon, a Republican, claims that he has the support of 55% of all voters in the 23rd U.S. Congressional District. Can the Central Committee conclude that less than 55% of all voters support Mr. Dixon, if, out of a random sample of 500 registered voters, only 245 expressed their preference for Mr. Dixon? Use $\alpha = 0.01$ as the level of significance.

Soln. Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: p = 0.55 \quad \text{vs} \quad \mathbf{H}_a: p < 0.55$$

Step2:Write the given information using proper symbols.

n=500, $\widehat{P} = \frac{245}{500} = 0.49$, $\widehat{P} \sim N\left(P_0\ ,\ \sqrt{\frac{P_0(1-P_0)}{n}}\right) \Rightarrow \widehat{P} \sim N\left(0.55\ ,\ \sqrt{\frac{0.55(1-0.55)}{500}} = 0.0222\right)$, α=.01

Test statistic. Z = -2.69662 (From TI-83)

Step3: Find the P-value. If the **P-value** is **less than α**, Reject $\mathbf{H}_0$.

P-value = 0.0035 (From TI-83)

Step4: Conclusion. Since the P-value=.0035 is less than α=.01, we reject the null hypothesis and the Central Committee conclude that less than 55% of all voters support Mr. Dixon.

Example 7. The sponsor of a weekly television show believes that the studio audience consists of an equal number of men and women. Out of 400 persons attending the show on a given night, 220 are men. Using a level of significance of 0.03, can we conclude that more than 50% of the people attending the show are men?

Soln. Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: p = 0.5 \quad \text{vs} \quad \mathbf{H}_a: p > 0.5$$

Step2:Write the given information using proper symbols.

n=400, $\widehat{P} = \frac{220}{400} = 0.55$, $\widehat{P} \sim N\left(P_0\ ,\ \sqrt{\frac{P_0(1-P_0)}{n}}\right) \Rightarrow \widehat{P} \sim N\left(0.5\ ,\ \sqrt{\frac{0.5(1-0.5)}{400}} = 0.025\right)$, α= 0.03

Test statistic. Z = 2 (From TI-83)

Step3: Find the P-value. If the **P-value** is **less than α**, Reject $\mathbf{H}_0$.

P-value = 0.0228 (From TI-83)

Step4: Conclusion. Since the P-value 0.0228 is less than α=.03, we reject the null hypothesis and conclude that more than 50% of the people attending the show are men?

Homework-Section 10.4: 13, 14, 16, and 20 page 499 (For these problems do hypothesis testing and draw a conclusion using the TI-83/84)