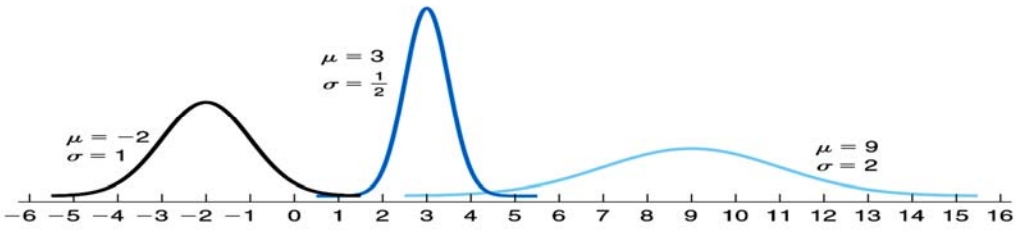
Chapter 7 -- The Normal Distribution

Sections 7.1, 7.2, and 7.3

In the world around us, we observe a wide variety of variables – height, weight, test scores, life of a brand of batteries, etc. Surprisingly, these variables share an important characteristic: their distributions have roughly the shape of a bell-shaped curve (normal curve) like the ones shown below.



A continuous random variable X is said to be **normally distributed** or to have a normal distribution if its distribution has the shape of a normal curve. Notation: X~N(μ, σ).

A normal distribution is completely determined by its mean and standard deviation. The probability density function of a normal random variable X is of the form $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$.

How to find probabil. for Any Normally Distrib. RV: using the TI-83/84?

Standard Normal Random Variable A normal random variable with a mean of 0 and a standard deviation of 1. Such a variable is denoted by Z. That is, Z ~ N(0, 1).

Example 1: Let Z ~ N(0, 1). Find the following probabilities:

The TI-83 command is: $Normalcdf\left(\begin{smallmatrix}Lower\\Bound\end{smallmatrix}, \begin{smallmatrix}Upper\\Bound\end{smallmatrix}, \mu, \sigma\right)$

1. $P(Z \le -1.15)$= Normalcdf(-E99, -1.15, 0, 1)= 0.1251
2. $P(Z \ge 1.15)$ = Normalcdf(1.15, E99, 0, 1)= 0.1251
3. $P(0 \le Z \le 0.83)$= Normalcdf(0, 0.83, 0, 1)= 0.2967
4. $P(-2.45 \le Z \le 1.36)$ Normalcdf(-2.45, 1.36, 0, 1)=0.9060
5. $P(Z \ge -0.34)$= Normalcdf(-0.34, E99, 0, 1)= 0.6331
6. P(Z < -1.5 or Z > 1.5) =1 - Normalcdf(-1.5, 1.5, 0, 1)= 0.1336

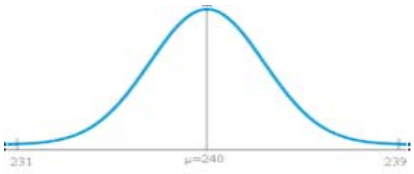
Example 2: Let Z ~ N(0, 1). Find the value of Z.
The TI-83 command is: $Invnormal(probability, \mu, \sigma)$

1. Find the Z such that the area to its left is 0.0594; Invnormal(0.0594, 0, 1)= -1.56
2. Find the Z such that the area to its right is 0.20; Invnormal(1-0.20, 0, 1)= 0.84

Find the value of Z_α if $\alpha = 0.05$; i.e. Find $Z_{0.05}$; Invnormal(1-0.05, 0, 1)= 1.6448

Example 3: Let X ~ N(240, 3). Find the following probabilities:

1. $P(X \le 200)$=normalcdf(-E99, 200, 240, 3)
2. $P(X \ge 300)$ =normalcdf(300, E99, 240, 3)
3. $P(200 \le X \le 300)$=normalcdf(200, 300, 240, 3)



Example 4: The personnel manager of a large company requires job applicants to take a certain test and achieve a score of at least 500. If the test scores are normally distributed with a mean of 485 and standard deviation of 30, what percentage of the applicants pass the test? What is the 90th percentile? (a) normalcdf(500, E99, 485, 30)=0.3085 (b) invnormal(.9, 485, 30)= 523.45

Example 5: Experience indicates that the development time for a photographic printing paper is normally distributed with a mean of 30 seconds and standard deviation of 1.1 seconds. Find the probability that it will take between 28.5 and 31.2 seconds for a randomly selected piece of photographic printing paper to develop. normalcdf(28.5, 31.2, 30, 1.1)= 0.7752

Example 6: The manufacturing process of ball bearings is Normally distributed with a mean diameter of 3 mm and a standard deviation of 0.1 mm. A customer has specification that requires that ball bearings have diameter between 2.85 and 3.1 mm. (a) What fraction of ball bearings manufactured meet specifications? (b) What fraction of ball bearings manufactured do not meet specifications? (a) normalcdf(2.85, 3.1, 3, 0.1) = 0.7745, (b) 1 – 0.7745 = 0.2255

Example 7: The test scores in Math 2620 are Normally distributed with mean of 60 and standard deviation 8. (a) What is the probability that a selected student has a score less than 73? (b) More than 56? (c) and (d) The instructor would like to assign F's to the lowest 15% of the class and A's to the top 5% of the class. What is the cut off point? (a) normalcdf(-E99, 73, 60, 8) =.9479, (b) normalcdf(56, E99, 60, 8) =.6915, (c) invnormal(.15, 60, 8) = 51.71-F, (d) invnormal(1-.05, 60, 8)= 73.16-A

Homework: Section 7.2: 13, 14, 15, 19, 20, 21, and 27 through 44 all pages 346-348.
Section 7.3 3, 5, 7, 9, 13, 14, 15, 16, 21, 23, 24, 25, 26 27 pages 354-355.