Chapter 6
Discrete Probability Distributions

Section 6.1 Discrete Random Variables

Random Variable (RV): A random variable assigns numerical value to each experimental outcome in the sample space.

Discrete Random Variable (DRV): A random variable that assumes only a finite number of values in an interval.

Continuous Random Variable (CRV): A random variable that assumes infinitely many number of values in an interval.

Example 1: Is it discrete or continuous random variable (rv):

(a) # of defective vending machines at VSU;
(b) The amount of time required to complete your homework each day for your math2620 class;
(c) Heights of male students enrolled at VSU in Fall 99;
(d) The number of students in your math class at VSU;
(e) Distance required to stop a car traveling at 70 mph;
(f) # of chess games you will have to play before winning a game of chess.

Probability Distribution: A listing of all possible values and corresponding probabilities of a discrete random variable.

Example 2: Toss two coins. Let X = "# of heads." List the values of X and the associated probabilities. Soln. S = {HH, HT, TH, TT}

X	f(x)	Outcome(s)
0	¼	TT
1	2/4	HT, TH
2	¼	HH

Note: **f(x)** above is called probability density function(pdf) for X.

Discrete Probability Distributions. Let X be a discrete random variable. For the function f(x) to be a pdf for X, the following two conditions must be satisfied:

1. $\sum P(x) = 1$
2. $0 \le P(x) \le 1$

Example 3: Given the probability distribution of a discrete random variable Y, find (a) P(Y= 2) (b) P($1 < Y \leq 4$) (c) P(Y < 5).

y	0	1	2	3	4	5
f(y)=P(Y=y)	0.1	0.2	?	0.3	0.25	0.1

Mean and Variance for a Discrete RV X

Expected Value of X (Mean): $E(X) = \mu = \sum(xf(x))$ or $\sum(xP(x))$

Variance: $\sigma_x^2 = VAR(X) = \sum(x-\mu)^2 P(x)$

$\sigma_x^2 = VAR(X) = \sum(x^2 P(x)) - \mu^2$ (computational formula)

Standard Deviation: $\sigma_x = S.D.(X) = \sqrt{Variance}$

Example 4: Reference Example 2, find E(X) and σ.
Soln.

X	P(x)	xP(x)	$x^2 P(x)$
0	¼	0	0
1	2/4	2/4	2/4
2	¼	2/4	4/4
		ΣxP(x)= 1	6/4 =Σ x^2 P(x)

$\mu = E(X) = \sum(xP(x)) = 1$

Note: The expected value of X should be interpreted as the long run average value of X.

$\sigma_x^2 = VAR(X) = \sum(x^2 P(x)) - \mu^2 = 6/4 - (1)^2 = 0.5$

$\sigma_x = S.D.(X) = \sqrt{0.5} = 0.71$

Example 5: Roll two dice. Let X = "sum of the two numbers." Find the probability function of X. Then find E(X) and S.D.(X).

Soln.

x	P(x)	xP(x)	x^2 P(x)
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
		ΣxP(x)= 252/36	1974/36 =Σx^2 P(x)

$\mu = E(X) = \sum(xP(x)) = 252/36 = 7$

$\sigma_x^2 = VAR(X) = \sum(x^2P(x)) - \mu^2 = 1974/36 - 49 = 54.83 - 49 = 5.83$

$\sigma_x = S.D.(X) = \sqrt{5.83} = 2.42$

HW: 7, 11, 12, 13, 14, 17, 18, 23(a, c , d only), 26(a, c, d only), 29, 31, 33 pp. 300-303.

Section 6.2 The Binomial Distribution

Properties of a Binonial Experiment

1. The experiment consists of a sequence of n identical and independent trials, that is, repeat a process n times.
2. There are two outcomes possible on each trial -- success or failure.
3. p = P[success] remains constant from trial to trial.

The general notation for a Binomial r.v. is **X~B(n,p).**

The probability density function(pdf) or probability distribution function (pdf) of the r.v. X is given by $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$; x=0,1, 2, …, n where

$\binom{n}{x} = nCx = \frac{n!}{x!(n-x)!}$.

Example 1: Flip a coin twice. Let the r.v. X be the number of heads. Does the r.v. X follow a binomial distribution? (Yes) **X~ B(2, 0.5)**

The pdf is: $f(x) = \binom{2}{x}(0.5)^x(1-0.5)^{2-x}$; where x = 0, 1, 2

The probability distribution for this experiment is:

X	0	1	2
P(X=x)	.25	.5	.25

Let us verify the pdf using the TI-83. The TI-83 Command is: $binompdf\left(n,\ p,\ {}^{Value}_{of\ X}\right)$.

Example 2: A baseball player with a batting average 30% comes to bat four times in a game. What is the probability he will hit the ball. (a) $P(X=0)$, (b) $P(X=1)$, (c) $P(1< X \le 2)$, (d) $P(X \ge 3)$.

(a) $P(X=0)$ = binompdf(4 , .3, 0) = 0.2401 (Using TI-83)
(b) $P(X=1)$ = binompdf(4 , .3, 1) = 0.4116 (Using TI-83)
(c) $P(1<X \le 2)$ = $P(X=2)$ = binompdf(4 , .3, 2) = 0.2646 (Using TI-83)
(d) $P(X \ge 3)$ = 1 - $P(X \le 2)$ =1- binomcdf(4 , .3, 2) = 0.0837 (Using TI-83)

Example 3: Do Example 3 on page 308 .

Example 4: Do Example 5 on page 311 .

Binomial Mean and Variance

If the r.v. X follows a binomial distribution with parameters **n** and **p**, **X ~ B(n,p)**, then the expected value and S.D. of X are give by

$\mu = E(X) = np,\ \sigma = \sqrt{npq} = \sqrt{np(1-p)}$

In example 5. X ~ B(2, .5). $\mu = E(X) = n.p = (2)(0.5) = 1.$

$\sigma^2 = n.p.q = n.p.(1-p) = 2(.5)(.5) = 0.5$

$\sigma = \sqrt{.05} = 0.7071$

Homework: 7, 8, 9, 10, 17, 18, 19, 23, 24, 25, 26, 27, 36, 37, 45, 46 pp. 315-316