Chapter 5
Probability and Random Variables

Section 5.1 Basic Concepts of Probability

Experiment: An act or process that generates well-defined outcomes.

Example:

1. Toss a coin.
2. Roll a die.
3. Selecting a random sample of size 2 from a group of five.

Sample Space: The collection of all possible outcomes of an experiment.

Simple Event: An individual outcome to an experiment.

Event: a subset (part) of the sample space.

Now we wish to assign probabilities to experimental outcomes. There are two approaches that are used most frequently.

1. The Classical Approach
2. Empirical or The Relative Frequency Approach

Regardless of the method used, the probabilities assigned must satisfy two basic requirements:

1. The probability assigned to each experimental outcome E must be between 0 and 1. That is,

$$0 \le P(E) \le 1 \ .$$

2. If $S = \{e_1, e_2, \ldots, e_n\}$, then $P(e_1) + P(e_2) + \ldots + P(e_n) = P(S) = 1$. If all the probabilities (positive) in the sample space sum to one then we have a **PROBABILITY MODEL.**

<u>Note:</u> The probability of an event is the sum of the probabilities of the simple events which comprise it.

Classical Approach: Assume that a given experiment has n different outcomes, each of which has an equal chance of occurring. Then,

Probability of each outcome = P(each outcome) = $\frac{1}{n}$.

<u>Examples 1 page 224:</u>

1. Toss a six-sided die. (a) Identify the outcomes of the probability experiment. (b) Determine the sample space. (c) Define the event E =”roll an even number”. (a) e_1=”rolling a 1” , e_2=”rolling a 2”, …, e_6=”rolling a 6” , (b) S = {1, 2, 3, 4, 5, 6}, (c) E=(2, 4,6}

2. Toss two coins. List the elements of the sample space and find P(“At least one head”).

3. Toss three coins. List the elements of the sample space and find the probability of the following events:

   a. A: “(exactly) two heads”
   b. B: “at least two heads”
   c. C: “at most two heads”

   Soln: P(A) = 3/8, P(B) = 4/8, P(C) = 7/8

4. Roll two dice. Find the probability of the following events:

   a. A: “Sum = 7”
   b. B: “Sum = 11”
   c. C: “Sum = 12”
   d. D: “Sum < 5”

Sum	Prob.
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

Soln: P(A) = 6/36, P(B) = 2/36, P(C) = 1/36, P(D) = 6/36 .

Empirical or Relative Frequency Approach: The relative frequency approach of assigning probabilities is appropriate when the data are available to estimate the proportion of the time the “outcome” will occur when the experiment is repeated a large number of times. Note that this approach does not require that each experimental outcome is equally likely.

Method for approximating P(E): Conduct (or observe) an experiment a large number of times and count the # of times the event E actually took place.

$$P(E) \approx \frac{\#\ of\ \ times\ E\ occurred}{\#\ of\ \ times\ experiment\ was\ repeated}$$

Note: As the experiment is repeated again and again, the empirical probability of success tend to approach the actual probability.

Example 5. The final exam in a course resulted in the following grades

Grade	A	B	C	D	F
Number	7	12	16	5	3

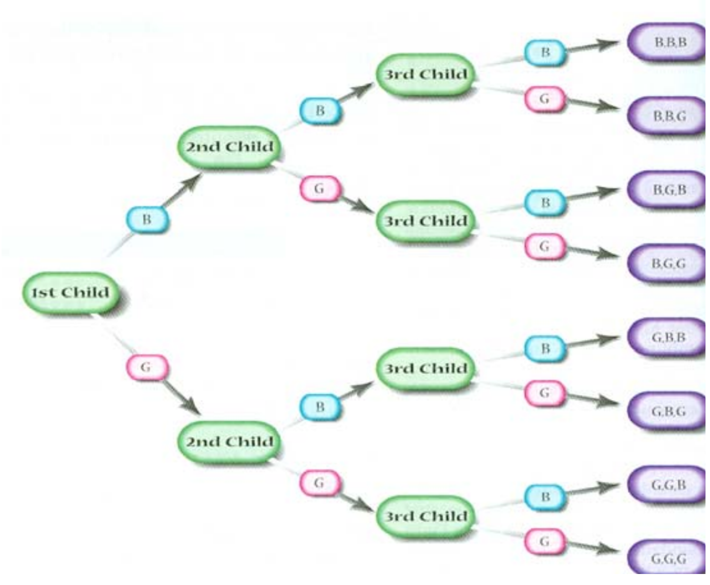
a. What is the probability that a randomly selected student received an A?

b. Verify that it's a probability model. $\frac{7}{43}+\frac{12}{43}+\frac{16}{43}+\frac{5}{43}+\frac{3}{43}=\frac{43}{43}=1$

Example 6 (Do Example 2 page 225)

Definition: An **unusual event** is an event that has low probability of occurring.

Example 7 on page 229 : Comparing the classical Method and Empirical Method.



Homework(Sect-5.1) 11-14 all, 22, 24, 26, 27, 29, 33, 39, and 52 pp 233-236.

Sections 5.2 Addition Rule and Complements

Union

The union of two events A and B is the event containing all sample points in A or B or both. **Notation: $A \cup B$**.

Intersection

The intersection of two events A and B is the event composed of all sample points that are in both A and B. **Notation: $A \cap B$.**

<u>**Note:**</u> $P(A \cup B) = P(A \text{ or } B)$ = P(event A occurs or event B occurs or they both occur)

$P(A \cap B) = P(A \text{ and } B)$ = P(event A and B both occur)

Complement of an Event: If A is an event over the sample space S, the complement of A **(Notation: A^C)** is defined to be the event consisting of all sample points in S that are not in A.

Subtraction Rule: $P(A) + P(A^C) = 1$

That is, $P(A^C) = 1 - P(A)$ or $P(A) = 1 - P(A^C)$

Example 1: Experiment. Roll a die once. S = { 1, 2, 3, 4, 5, 6} .

We defined the following events: A = {2, 4, 6} . P(A) = 3/6

B = {1, 3, 5 }. P(B) = 3/6

C = {1, 2, 3, 4} . P(C) = 4/6

$A\cup B$ = { 1, 2, 3, 4, 5, 6} $P(A\cup B)$ = 1/6+ 1/6+1/6+1/6+1/6+1/6= 6/6 = 1.

$A\cup C$ = { 1, 2, 3, 4, 6 } $P(A\cup C)$ = 1/6+ 1/6+1/6+1/6+1/6 = 5/6.

$B\cup C$ = { 1, 2, 3, 4, 5 } $P(B\cup C)$ = 1/6+ 1/6+1/6+1/6+1/6 = 5/6.

$A\cap B = \varnothing$ (Null set} $P(A\cap B)$ = 0.

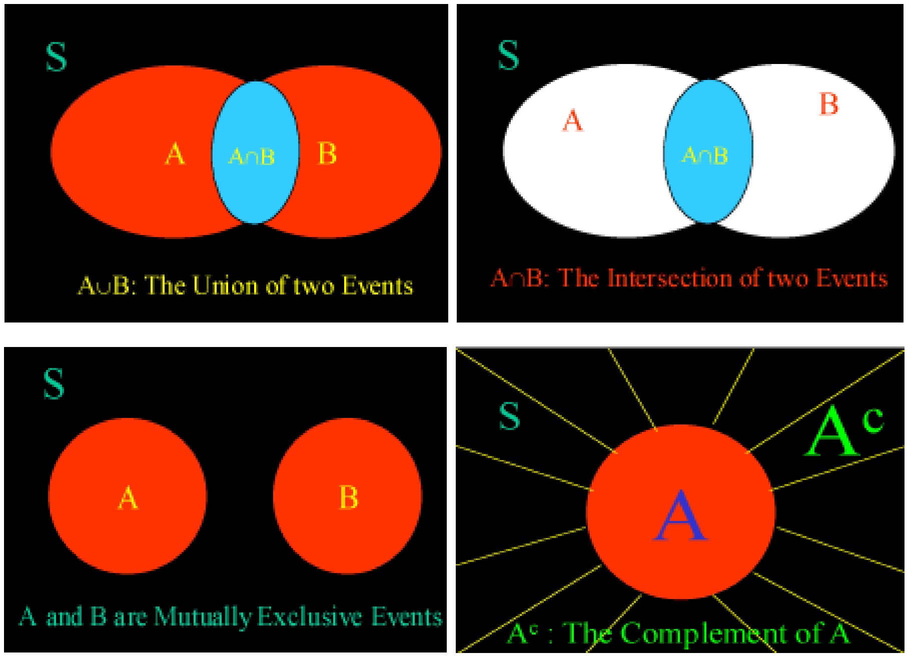
$A\cap C$ = { 2, 4 } $P(A\cap C)$ = 2/6.

$B\cap C$ = { 1, 3 } $P(B\cap C)$ = 2/6.

A= { 2, 4, 6 } A^c = { 1, 3, 5 } $P(A^c)$= 3/6 = 1 - P(A)= 1 - 3/6 = 3/6

NOTE: $A\cup A^c$ = S (sample space) and $A\cap A^c = \varnothing$.

Using Venn Diagrams: Union, Intersection, and Complement



Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 2: In a study of 100 students that had been awarded university scholarships, it was found that 40 had part-time jobs, 25 had made the dean's list the previous semester, and 15 had both a part-time job and had made the dean's list. What was the probability that a student had a part-time job or was on the dean's list?

Soln: $P(A \cup B)=P(A)+P(B)-P(A \cap B)=(40/100)+(25/100)-(15/100)=(50/100)= 0.5$

Example 3: You are playing a card game in which spades and honors (Ace, Queen, King, or Jack) are valuable. If you draw a card from the full deck, what is the probability that it is a "valuable card?" Soln: 25/52

Mutually Exclusive Events: ("ME") Two events A and B are called mutually exclusive if $A \cap B$ contains no sample points. That is, A and B have no outcomes in common.

Note: If A and B are mutually exclusive, then $P(A \cap B) = 0$.

Addition Rule for "ME" Events

If A and B are "ME", then $P(A \cup B) = P(A) + P(B)$

<u>Do Example 4 on page 242</u>

		Gender		
		Male	Female	Totals
Marital Status	Never Married	30.3	25.0	55.3
	Married	63.6	64.1	127.7
	Widowed	2.6	11.3	13.9
	Divorced	9.7	13.1	22.8
	Totals	106.2	113.5	219.7

Solution: Get the probability table and finish it in class.

		Gender		
		Male	Female	Totals
Marital Status	Never Married	0.138	0.114	0.252
	Married	0.289	0.292	0.581
	Widowed	0.012	0.051	0.063
	Divorced	0.044	0.060	0.104
	Totals	0.483	0.517	1

Example 4: If one card is drawn from a full deck of 52 cards, what is the probability that the card drawn is a queen or a six? Soln. 2/13

Example 5: If you roll a pair of dice once, what is the probability of getting a one on the first die or the second die ? Soln: 11/36

Example 6: Let M=person interviewed is male
F=Person Interviewed is female
D=Person believes the man should pay on first date
W=Person believes the one who requests the date should pay
E=Person believes the one who earns more money should pay on first date
As reported in *Money* magazine (November 1998), P(M)=.61, P(D)=.41, P(W)=.41, P(E)=.02, and $P(D \cap M)=.29$

a) Are M and D mutually exclusive events
b) Find $P(D \cup M)$
c) Find $P(D \cup W)$
d) Find $P(D \cap F)$
e) Find $P(D \cup F)$
Soln: a) NO b) 0.73, c) 0.82 d) 0.12 e) 0.68

Homework: Section 5.2 5, 6, 7, 9, 12-19 all 31, 38, and 41 pp 245-248.

Section 5.3 Independence and the Multiplication Rule

Independent Events: Two events A and B are called independent if the occurrence of one does not affect the probability of the occurrence of the other.

Multiplication Rule for independent events
P(A and B) = P(A)P(B) also $P(A \cap B) = P(A)P(B)$

Example 7: Flip a coin twice. The probability of finding two heads is 0.25 since the sample space is S = {HH, HT, TH, TT}; i.e P(HH)= 0.25.

Now, we can use the multiplication rule for independent events to show that the P(HH)=0.25. $P(H \cap H) = P(H)P(H)=(0.5)(0.5)=0.25$.

<u>**D0 Example 3 on page 252:**</u> Life Expectancy

<u>**D0 Example 4 on page 253:**</u> Computing At-Least Probabilities using the complement

Homework: Section 5.3 1- 6 all, 11, 12, 25, 27, and 30 pp 254-255.

Section 5.4 Conditional Probability:

$$P\langle A|B\rangle = \frac{P(A\cap B)}{P(B)} \quad \textbf{or} \quad P\langle B|A\rangle = \frac{P(A\cap B)}{P(A)}$$

Example 8: Roll a die. Let A = an the number is 3, and B = the number is Odd . Find a) P(A) b) P(B) c) P(A ∩ B) d) P(A|B)

Soln. a) 1/6 b) 3/6 c) 1/6 d) 1/3

Example 9: A major metropolitan police force in the eastern United States consists of 1200 officers, 960 men and 240 women. Over the past 2 years, 324 officers on the police force have been awarded promotions. The breakdown of the promotions is given in the following table. A committee of female officers charged discrimination. Are they right or wrong?

		Gender		
		Male	Female	Totals
Promotion	Promoted	288	36	324
	Not Promoted	672	204	876
	Totals	960	240	1200

Solution: Get the probability table and finish it in class.

		Gender		
		Male	Female	Totals
Promotion	Promoted	.24	.03	.27
	Not Promoted	.56	.17	.73
	Totals	.80	.20	1

General Multiplication Rule (When the events are **Dependent**)

P(A ∩ B) = P(A)P(B|A) or P(A ∩ B) = P(B)P(A|B)

Multiplication Rule (When the events are **Independent**)

P(A ∩ B) = P(A)P(B)

Example 10: A service station manager knows from past experience that 80% of the customers use a credit card when purchasing gasoline. What is the probability that the next two customers will both pay with a credit card? Soln: 0.64

Example 11: I would like to purchase my dream house. My chance of getting promoted is 60%.   The probability of buying the house if I get promoted is 80%. What is the probability of getting promoted and buying the house?   Soln: 48%

Homework: 1, 2 , 3, 7, 8, 18, 19, 24, 26, 27, and 28   pp. 263-264.

Additional questions for 18 and 19.
18- Is Cancer independent of Cigar Smoking?
19- Is Traffic fatalities independent of the Gender?