Chapter 11
Comparing Two Population Means

Section 11.2 In this chapter, we will compare the means of two populations. For example,

a. Does Advil fight pain better than Tylenol?
b. Are men older than women when they graduate from college?
c. Are SAT scores higher after taking a preparation course than before?

If the two populations are Normally distributed or the sample sizes are sufficiently large ($n_1 \geq 30$ *and* $n_2 \geq 30$) then $\bar{X}_1 - \bar{X}_2 \sim t_O$ where t_O is computed using the smaller of $n_1 - 1$ and $n_2 - 1$. The confidence interval for μ_1 - μ_2 has the form: $(\bar{X}_1 - \bar{X}_2) \pm t_O \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$. Use the TI 83/84 to compute it.

Example 1. The same standardized test is given to students selected at random from two schools. The following table summarizes the results:

School A: sample size = 50, mean = 68.7, sd = 6.0
School B: sample size = 40, mean = 65.1, sd = 6.4
Construct a 95% C.I. estimate for μ_1 - μ_2.

Solution: (0.973, 6.228) 95% of the time School A scores 1 to 6 points higher than School B. (Use TI-83)

Example 2. A small amount of selenium, from 50 to 200 micrograms per day, is considered essential to good health. Suppose that two random samples of 30 adults were selected, one from region A and another from region B of the United States. A day's intake of selenium from both, liquids and solids, was recorded for the 30 adults from region A and region B. The following was observed:

Region A: mean = 167.1, sd = 24.3 micrograms.
Region B: mean = 140.9, sd = 17.6 micrograms.

Construct a 90% C.I. estimate for μ_1 - μ_2.
Solution: (17.03, 35.37) (Use TI-83) We are 90% confident that region A has 17.03 to 35.37 micrograms of selenium higher than region B.

Example 3. A survey of recent graduates with degrees in Marketing and Accounting revealed the following starting salaries in thousands of dollars. Assume both populations are normally distributed.

Marketing: 24.6, 27.8, 25.8, 23.1, 25.2 24.7, 26.1, 22.6, 23.8
Accounting: 26.7, 24.9, 27.1, 23.1, 27.5, 27.4, 24.9, 26.3, 28.9, 26.4, 28.1, 29.7, 25.5, 24.9, 28.5

Build a 97% C.I. for the mean difference in the starting salaries of Accounting and Marketing majors.

Solution: (-3.465, -0.144) (Use a TI-83) We are 97% confident that the Marketing salaries are $114 to $3,465 lower than the Accounting

Hypothesis Testing for $\mu_1 - \mu_2$

Example 4. A sampling of 50 women and 40 men produced a mean age at death for women of 75.5 years and for men of 67.9 years with a sd of 16.2 and 18.3 respectively. At the 0.05 level of significance, do women live longer than men?

Solution. Let μ_1 = mean age for women at death.
μ_2 = mean age for men at death.

Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: \mu_1 - \mu_2 = 0 \quad \text{vs} \quad \mathbf{H}_a: \mu_1 - \mu_2 > 0$$

Step 2: Write the given information.

$\bar{X}_1 = 75.5$, $n_1 = 50$, $S_1 = 16.2$, $\bar{X}_2 = 67.9$, $n_2 = 40$, $S_2 = 18.30$

Find the test statistic. $t_O = t_{39}$ 2.06

Step 3: Find the p-value: p= 0.021

Step 4: Conclusion. Since the p=.021 < α=.05, reject the null hypothesis and conclude that women live longer than men.

Example 5. Last year a sample of 100 new cars manufactured by a Japanese car-maker averaged 33.8 mpg. This year a sample of 100 new cars of the same car-maker averaged 35.1 mpg. If each sample had a sd of 10 mpg, can the car-maker conclude that they have significantly increased the performance their cars? Use 0.01 as the level of significance.

Solution. Let μ_1 = average mpg of last year's cars.
μ_2 = average mpg of this year's cars.

Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: \mu_1 - \mu_2 = 0 \quad \text{vs} \quad \mathbf{H}_a: \mu_1 - \mu_2 < 0$$

Step 2: Write the given information..

$\bar{X}_1 = 33.8,\ n_1 = 100,\ S_1 = 10 \qquad \bar{X}_2 = 35.1,\ n_2 = 100,\ S_2 = 10$

Find the test statistic. $t_O = t_{49} = -0.92$

Step 3: Find the p-value: p= 0.179

Step 4: Conclusion. Since the p=0.179 > α=.01, we fail to reject the null hypothesis and conclude that they have not significantly increased the performance of their cars.

Example 6. A company assembles large electronic devices. The assembly method they use is rather simple and has been used for a long time. Recently, the company has become aware of a new assembly technique. The company is basically satisfied with their current method of assembly. However, if the company finds that the new method significantly reduces assembly time, they would adopt this new method. The company decides to do a test to compare the two methods over the course of a day. One production line will use the current method and the other production line will try the new method. The data (in minutes) are shown below:

Current: 37.3, 55.9, 45.4, 52.2, 39.1, 34.4

New : 47.4, 57.3, 39.8, 60.4, 46.7, 36.4

Should the company adopt the new method? Use α=.10 .

Solution. Let μ_1 = mean assembly time with the current method.

μ_2 = mean assembly time with the new method.

Step1: State the null and alternative hypotheses.

$$\mathbf{H}_0: \mu_1 - \mu_2 = 0 \quad \text{vs} \quad \mathbf{H}_a: \mu_1 - \mu_2 > 0$$

Step 2: Write the given information.

$\bar{X}_1 = 44.05,\ n_1 = 6,\ S_1 = 8.62,\ \bar{X}_2 = 48.00,\ n_2 = 6,\ S_2 = 9.42,\ \alpha = 0.1$

Find the test statistic. $t_O = t_5 = -0.7416$

Step 3: Find the p-value: p= 0.762

Step 4: Conclusion. Since the p=0.762 > α=.10, We fail to reject the null hypothesis and conclude that the assembly time is the same for both. Therefore, the company should not adopt the new method.

HW: 3, 5, 6, 10, 11(c, d only), 12(c, d only), 15, 16(c only) pp. 529-531.